Is ``just-so'' Higgs splitting needed for $t-b-\tau$ Yukawa unified SUSY GUTs?

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ABSTRACT: Recent renormalization group calculations of the sparticle mass spectrum in the Minimal Supersymmetric Standard Model (MSSM) show that $t - b - \tau$ Yukawa coupling unification at M_{GUT} is possible when the mass spectra follow the pattern of a radiatively induced inverted scalar mass hierarchy. The calculation is entirely consistent with expectations from SO(10) SUSY GUT theories, with one exception: it seems to require MSSM Higgs soft term mass splitting at M_{GUT}, dubbed "just-so Higgs splitting" (HS) in the literature, which apparently violates the SO(10) gauge symmetry. Here, we investigate three alternative effects: *i*). SO(10) *D*-term splitting, *ii*). inclusion of right hand neutrino in the RG calculation, and *iii*). first/third generation scalar mass splitting. By combining all three effects (the DR3 model), we find $t - b - \tau$ Yukawa unification at M_{GUT} can be achieved at the 2.5% level. In the DR3 case, we expect lighter (and possibly detectable) third generation and heavy Higgs scalars than in the model with HS. In addition, the light bottom squark in DR3 should be dominantly a right state, while in the HS model, it is dominantly a left state.

Contents

1   Introduction

The four LEP experiments performed precision measurements of the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge coupling constants of the Standard Model (SM) at energy scale $Q = M_Z$ [1]. It is an astonishing fact that the values of these couplings, evolved in energy from the weak scale to an energy scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV, nearly meet at a point under Minimal Supersymmetric Standard Model (MSSM) renormalization group (RG) evolution [2], while their unification fails badly under Standard Model RG evolution. This latter fact is often touted as indirect evidence that the MSSM (with weak-scale sparticle masses) is the correct effective field theory describing nature at energy scales between M_{GUT} and M_{weak}, and further that nature may well be described by a supersymmetric grand unified theory (SUSY GUT) at energy scales above M_{GUT}.

While the gauge group SU(5) early on emerged as a leading GUT group candidate [3], the gauge group SO(10) appears to be much more compelling [4]. In SUSY SO(10) GUTs, a number of attractive features emerge.

- All matter fields of a single generation are unified into the 16-dimensional spinor of SO(10): thus, SO(10) unifies matter as well as forces.

- The seemingly ad-hoc cancellation of triangle anomalies in the SM and in SU(5) is a simple mathematical fact in SO(10).

- The 16th element of the SO(10) matter spinor is naturally occupied by a superfield $\hat{N}^c$ which contains a SM-gauge singlet right-handed neutrino (RHN) field. Upon breaking of SO(10), the RHN acquires a Majorana mass M_N which leads to the famous see-saw relation for neutrino masses [5]: $m_\nu = (f_\nu v_u)^2/M_N$.

- The fact that matter superfields lie in a spinor representation automatically leads to R-parity conservation, since only superpotential terms of the form *matter-matter-Higgs* are allowed by the SO(10) symmetry, while the R-parity violating *matter-matter-matter* or *matter-Higgs* products are not allowed. While SO(10) breaking may re-introduce R-parity violation, many simple breaking schemes exactly preserve the R-parity conserving structure.

- SO(10) SUSY GUTs naturally explain why two Higgs doublets occur in nature at the weak scale. The **2** and **2*** MSSM Higgs doublets lie in a **5** and **5*** of SU(5), and the **10** of SO(10) naturally contains a **5** and **5*** under restriction to SU(5).

Thus, SO(10) SUSY GUTs provide an extremely compelling picture of what physics might look like around the GUT scale.

Along with gauge coupling and matter unification, the simplest SO(10) SUSY GUT models also predict third generation $t - b - \tau$ Yukawa coupling unification at the GUT scale [6][1]. To check this assertion, one must begin with the measured third generation fermion masses — m_t, m_b and m_τ — and calculate the associated Yukawa couplings at the weak scale. Then one may evolve the $t - b - \tau$ Yukawa couplings up in energy to check whether they unify at $M_{\rm GUT}$, just as the gauge couplings do. The values of the weak scale Yukawa couplings depend strongly on the ratio of Higgs field vevs: $\tan\beta \equiv v_u/v_d$. Furthermore, unlike the gauge couplings, the Yukawa couplings have a large dependence on sparticle loop corrections when passing from the SM to the MSSM effective field theories [7]. Thus, the $t - b - \tau$ Yukawa coupling unification depends on the precise form of the sparticle mass spectra of the MSSM. This latter fact offers a consistency check: if sparticle masses are found with the expected pattern, it would be strongly suggestive that an SO(10) SUSY GUT model is valid around the GUT scale.

Detailed calculations of when $t - b - \tau$ Yukawa couplings unify at $M_{\rm GUT}$ within the MSSM context have been performed by several groups [8–14]. They use an SO(10)-inspired model parameter space given by

$$m_{16}, \quad m_{10}, \quad M_D^2, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad {\rm sign}(\mu) \tag{1.1}$$

where m_{16} is the GUT scale mass of all matter scalars, m_{10} is the GUT scale mass of Higgs scalars, M_D parametrizes Higgs mass splitting (HS) or possible scalar mass D-term splitting (DT) (the latter can arise from the breaking of SO(10) gauge symmetry), $m_{1/2}$ is the unified GUT scale gaugino mass, A_0 is the unified GUT scale soft SUSY breaking (SSB) trilinear term, $\tan\beta \equiv v_u/v_d$ is the weak scale ratio of Higgs field vevs, and μ is the superpotential Higgs bilinear term, whose magnitude — but not sign — is determined by the electroweak symmetry breaking (EWSB) scalar potential minimization conditions.

In practice, the two Higgs field soft breaking terms — $m_{H_u}^2$ and $m_{H_d}^2$ — cannot be degenerate at $M_{\rm GUT}$ and still allow for an appropriate radiative breakdown of electroweak symmetry (REWSB) [15]. Effectively, $m_{H_u}^2$ must be less than $m_{H_d}^2$ at $M_{\rm GUT}$ in order to give $m_{H_u}^2$ a head start in running towards negative values at $M_{\rm weak}$. We parametrize the

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[1]M. Carena, M. Olechowski, S. Pokorski and C. Wagner, ref. [7];

Higgs splitting as $m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2$ in accord with nomenclature for D-term splitting to scalar masses when a gauge symmetry undergoes a breaking which reduces the rank of the gauge group [16]. While the D-term splitting should apply to matter scalar SSB terms as well, in practice, better Yukawa unification for $\mu > 0$ is found when the splitting is restricted only to the Higgs SSB terms. The mass splitting applied only to Higgs scalars, and not to matter scalars, has been dubbed "just-so" Higgs splitting in the literature [10].

In previous work, the above parameter space was scanned over via random scans [8, 11] and also by more efficient Markov Chain Monte Carlo (MCMC) scans [12] to search for Yukawa unified solutions using the Isasugra subprogram of ISAJET [17] for sparticle mass computations.[2] The quantity

$$R = \frac{max(f_t, f_b, f_\tau)}{min(f_t, f_b, f_\tau)} \quad \text{(evaluated at Q = M}_{\text{GUT}}\text{),} \tag{1.2}$$

was examined, where solutions with $R \simeq 1$ gave apparent Yukawa coupling unification. For superpotential Higgs mass parameter $\mu > 0$ (as favored by $(g-2)_\mu$ measurements), Yukawa unified solutions with $R \sim 1$ were found but only for *special* choices of GUT scale boundary conditions [8–12, 14, 19]:

- $m_{16} \sim 3 - 15$ TeV,

- $A_0 \sim -2m_{16}$,

- $m_{10} \sim 1.2m_{16}$,

- $m_{1/2} \ll m_{16}$,

- $\tan \beta \sim 50$.

Models with this sort of boundary conditions were derived even earlier in the context of radiatively driven inverted scalar mass hierarchy models (RIMH) which attempt to reconcile suppression of flavor-changing and CP-violating processes via a decoupling solution with naturalness via multi-TeV first/second generation and sub-TeV scale third generation scalars [20, 21]. The Yukawa-unified spectral solutions were thus found in refs. [11, 12] to occur with the above peculiar choice of boundary conditions as long as m_{16} was in the *multi*-TeV regime.

Based on the above work [11, 12], the sparticle mass spectra from Yukawa-unified SUSY models are characterized qualitatively by the following conditions:

- first and second generation scalars have masses in the ~ 10 TeV regime,

- third generation scalars, μ and m_A have masses in the few TeV regime (owing to the inverted scalar mass hierarchy),

- the gluino has mass $m_{\tilde{g}} \sim 300 - 500$ GeV,

- the lightest neutralino $\widetilde{Z}_1$ is nearly pure bino with mass typically $m_{\widetilde{Z}_1} \sim 50 - 80$ GeV.

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[2]Ref. [14] confirms the general structure of Yukawa-unified models also using the SoftSUSY [18] spectrum generator.

The presence of a bino-like $\widetilde{Z}_1$ along with multi-TeV scalars gives rise to a neutralino cold dark matter (CDM) relic abundance that is typically in the range $\Omega_{\widetilde{Z}_1} h^2 \sim 10^2 - 10^4$, i.e. far above [22] the WMAP measured [23] value $\Omega_{\rm CDM} h^2 = 0.110 \pm 0.006$ by several orders of magnitude. A very compelling solution to the Yukawa-unified dark matter abundance problem occurs if one invokes the Peccei-Quinn solution [24] to the strong CP problem, which leads to dark matter being composed of an axion [25, 26]/axino [27, 28] admixture [12, 14, 29], instead of neutralinos. The axino then can serve as the LSP instead of the lightest neutralino [30, 31]. Cosmological solutions with a re-heat temperature T_R high enough to sustain non-thermal leptogenesis ($T_R \sim 10^6 - 10^8$ GeV) could most easily be found if the dominant component of the cold dark matter consisted of *axions* rather than axinos.

The above calculational results show that Yukawa unified solutions are compatible with the MSSM as the effective theory between the GUT and weak mass scales, but for a very constrained form of the sparticle mass spectrum. Indeed, the entire scheme is compatible with expectations from an SO(10) SUSY GUT theory with GUT symmetry broken at the scale $M_{\rm GUT}$, save for one feature. Naively, SO(10) symmetry implies the two Higgs masses should be *degenerate* at $M_{\rm GUT}$, at tree-level: i.e. $m_{H_u}^2 = m_{H_d}^2$. The mechanism for the large GUT scale Higgs soft term splitting is unknown, and violates the SO(10) symmetry. There is of course a well-known source of Higgs soft term splitting: namely, the D-term contribution to scalar masses which is induced when the rank of the gauge group is reduced from 5 of SO(10) to 4 of SU(5) or the SM. The D-term contributions are necessarily proportional to the charge of the subgroup $U(1)_X$, and are to be included in *all* scalar fields carrying a $U(1)_X$ charge. At $Q = M_{\rm GUT}$, the gauge symmmetry breaking induces a scalar mass contribution

$$m_Q^2 = m_E^2 = m_U^2 = m_{16}^2 + M_D^2 \tag{1.3}$$
$$m_D^2 = m_L^2 = m_{16}^2 - 3M_D^2 \tag{1.4}$$
$$m_{\tilde{\nu}_R}^2 = m_{16}^2 + 5M_D^2 \tag{1.5}$$
$$m_{H_{u,d}}^2 = m_{10}^2 \mp 2M_D^2, \tag{1.6}$$

where the contribution M_D^2 is effectively a free parameter of order the weak scale, and whose value depends on the details of SO(10) breaking. It can take positive or negative values.[3] It was found in refs. [9–11] that calculationally, the spectral solutions with the best $t - b - \tau$ Yukawa unification occurred when the D-term splitting was applied only to the Higgs scalars, and not to the other matter scalars.

In this paper, we re-visit the question of the HS case versus the DT splitting case in $t - b - \tau$ Yukawa-unified models. We search for Yukawa-unified solutions while including several effects which are all consistent with the general framework of simple SO(10) SUSY GUT models: *i.*) application of full DT splitting to all scalar masses, *ii.*) inclusion of neutrino Yukawa coupling effects (RHN) [10, 21], and *iii.*) inclusion of mass splitting between the third generation, versus the first two generations of matter scalars (3GS). We scan over SO(10) model parameter space using the MCMC technique, which provides an efficient search for the best Yukawa unified solutions. We find that each of the above three

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[3]Sum rules for sparticle masses as a test of the underlying SO(10) are discussed in [32].

effects acts to improve the degree of Yukawa unification compared to results without the effects, but none of them work as well as the just-so HS model. However, using all three effects simultaneously (the DR3 model) does allow us to reach Yukawa-unified solutions with $R \sim 1.025$, i.e. Yukawa unification to below the 3% level. The remaining last few percent might then be compatible with expected GUT scale threshold effects (which are of course model dependent) and intrinsic theory error in our 2-loop RGE calculations. We find that the superparticle mass spectra using the DR3 model is somewhat modified from solutions using just-so HS. In particular, for a given value of m_{16}, the predicted value of m_A and $m_{\tilde{b}_1}$ are much lighter than in the just-so HS prediction. For $m_{16} \sim 10\,\text{TeV}$, m_A and $m_{\tilde{b}_1}$ can extend down to or even below the $1\,\text{TeV}$ level, and may be accessible to LHC searches and/or to searches at a future CERN e^+e^- Linear Collider (CLIC), where center-of-mass energies of order $\sqrt{s} \sim 3 - 5\,\text{TeV}$ are proposed. The spectral differences, and the expected left-right composition of the lightest bottom squark, may offer a means to distinguish between the just-so HS model and the DR3 model.

2 Higgs mass splitting and radiative EWSB in Yukawa unified models

2.1 Radiative EWSB in Yukawa unified models

MSSM models with $t - b - \tau$ Yukawa coupling unification and degenerate (Higgs) soft masses at $Q = M_{\text{GUT}}$ face a well-known problem when one attempts to generate realistic sparticle mass spectra: electroweak symmetry fails to be appropriately broken [6, 15]. The problem can be seen by examining the one-loop Higgs soft mass RGEs:

$$\frac{dm_{H_d}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 - \frac{3}{10}g_1^2 S + 3f_b^2 X_b + f_\tau^2 X_\tau\right), \tag{2.1}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{2}{16\pi^2}\left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t\right), \tag{2.2}$$

where

$$X_t = m_{Q_3}^2 + m_{\tilde{t}_R}^2 + m_{H_u}^2 + A_t^2, \tag{2.3}$$

$$X_b = m_{Q_3}^2 + m_{\tilde{b}_R}^2 + m_{H_d}^2 + A_b^2, \tag{2.4}$$

$$X_\tau = m_{L_3}^2 + m_{\tilde{\tau}_R}^2 + m_{H_d}^2 + A_\tau^2, \tag{2.5}$$

and

$$S = m_{H_u}^2 - m_{H_d}^2 + Tr\left[\mathbf{m}_Q^2 - \mathbf{m}_L^2 - 2\mathbf{m}_U^2 + \mathbf{m}_D^2 + \mathbf{m}_E^2\right]. \tag{2.6}$$

The right-hand side terms with negative co-efficients give an *upwards* push to the Higgs soft masses during evolution from M_{GUT} to M_{weak}, while the positive terms give a *downwards* push.

At the weak scale, where the Higgs effective potential is minimized, the EWSB minimization conditions require that (at tree-level)

$$B\mu = \frac{(m_{H_u}^2 + m_{H_d}^2 + 2\mu^2)\sin 2\beta}{2}, \tag{2.7}$$

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{(\tan^2\beta - 1)} - \frac{M_Z^2}{2}. \tag{2.8}$$

The first of these determines the weak scale value of B in terms of $\tan\beta$; the second relation determines the magnitude, but not the sign, of the superpotential μ term. At moderate-to-large $\tan\beta$ values and $|m^2_{H_u}| \gg M^2_Z$, the second relation also gives approximately $\mu^2 \simeq -m^2_{H_u}$, and we see that $m^2_{H_u}$ must be driven to negative values to accommodate successful REWSB. In models with Yukawa couplings $f_t > f_b$, f_τ, the $m^2_{H_u}$ term is pushed to negative values by the large value of f_t in eq. (2.2), resulting in successful EWSB. In contrast, in models with $t-b-\tau$ Yukawa unification, the Yukawa coupling terms on the right-hand side of the $m^2_{H_d}$ equation are *larger* than the corresponding terms in the $m^2_{H_u}$ equation, resulting in $m^2_{H_d}$ being driven more negative than $m^2_{H_u}$ at the weak scale. If $m^2_{H_d} < m^2_{H_u} \tan^2\beta$, then $\mu^2 < 0$, signaling an inappropriate EWSB. The solution to this dilemma so far in Yukawa unified models is to provide the $m^2_{H_u}$ term a *head-start* in running to negative values at the weak scale by adopting Higgs splitting such that $m^2_{H_u} < m^2_{H_d}$ at the GUT scale.

While just-so HS applies a splitting (that violates SO(10) gauge symmetry) only to Higgs soft masses, and leaves the remaining GUT scale scalar masses fixed at m_{16}, the expected splitting due to D-terms arising from SO(10) breaking at $M_{\rm GUT}$ apply to matter scalars as well as Higgs scalars as given in eqs. (1.3)–(1.6). The problem arising from DT splitting is that the m^2_D terms are substantially reduced already at $M_{\rm GUT}$, and can get driven tachyonic at $M_{\rm weak}$ through the RIMH mechanism. What is needed in terms of DT splitting is a large enough Higgs splitting to facilitate REWSB, but not so large a splitting that $m^2_{\tilde{b}_R}$ is driven tachyonic at $M_{\rm weak}$.

2.2 The role of neutrino Yukawa couplings

An alternative mechanism to aide $m^2_{H_u}$ being driven negative is to *balance* the right-hand-side Yukawa push in eqs. (2.1)–(2.2) by incorporating the effect of the third generation neutrino Yukawa coupling. In this case, the $m^2_{H_u}$ soft term RGE is modified to

$$\frac{dm^2_{H_u}}{dt} = \frac{2}{16\pi^2} \left[-\frac{3}{5} g_1^2 M_1^2 - 3 g_2^2 M_2^2 + 3 f_t^2 X_t + f_\nu^2 X_\nu \right] \tag{2.9}$$

with $X_\nu = m^2_L + m^2_{\tilde{\nu}_R} + m^2_{H_u} + A^2_\nu$, and the RGE for $m^2_{H_d}$ is unchanged. We see that for unified Yukawa couplings, now $m^2_{H_u}$ receives an *additional* downward push from the f_ν term [10, 21]. The f_ν term contributes to the running at energy scales $M_N < Q < M_{\rm GUT}$. For $Q < M_N$, the RHN states are integrated out of the effective theory, and the RGEs revert to the MSSM form of eq. (2.2).

2.3 Third generation splitting

A third effect that can improve the implementation of EWSB in Yukawa unified models is to allow for non-degeneracy in first/second versus third generation scalar masses: third generation mass splitting (3GS). In general, generational non-degeneracy can lead to flavor violation in excess of experimental bounds [33]. The bounds on flavor-changing neutral currents (FCNC) apply most strongly to splitting amongst the first and second generation soft terms; contraints on third generation FCNCs are much less restrictive [34]. Here, we will maintain degeneracy between $m_{16}(1)$ and $m_{16}(2)$, although some small breaking

in these terms is allowed, especially in our case where scalar masses are quite heavy and typically in the 10 TeV range, where we also have FCNC suppression via decoupling. Here, we will adopt $m_{16}(1) = m_{16}(2) \neq m_{16}(3)$.

At tree level, we would expect 3GS to not affect Yukawa coupling evolution and EWSB, since the Higgs sector only couples strongly to third generation scalars, and is independent of first/second generation scalar masses. However, at two-loop level, the scalar mass RGEs have the form given by [35]

$$\frac{dm_i^2}{dt} = \frac{1}{16\pi^2}\beta_{m_i^2}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{m_i^2}^{(2)}, \tag{2.10}$$

where $t = \ln Q$, $i = Q_j$, U_j, D_j, L_j and E_j, and $j = 1 - 3$ is a generation index. Two loop terms are formally suppressed relative to one loop terms by the square of a coupling constant as well as an additional loop factor of $16\pi^2$. However, these two loop terms include contributions from *all* scalars. Specifically, the two loop β functions include,

$$\beta_{m_i^2}^{(2)} \ni a_i g_3^2 \sigma_3 + b_i g_2^2 \sigma_2 + c_i g_1^2 \sigma_1, \tag{2.11}$$

where

$$\sigma_1 = \frac{1}{5}g_1^2\left\{3\left(m_{H_u}^2 + m_{H_d}^2\right) + Tr\left[\mathbf{m}_Q^2 + 3\mathbf{m}_L^2 + 8\mathbf{m}_U^2 + 2\mathbf{m}_D^2 + 6\mathbf{m}_E^2\right]\right\},$$
$$\sigma_2 = g_2^2\left\{m_{H_u}^2 + m_{H_d}^2 + Tr\left[3\mathbf{m}_Q^2 + \mathbf{m}_L^2\right]\right\}, \quad \text{and}$$
$$\sigma_3 = g_3^2 Tr\left[2\mathbf{m}_Q^2 + \mathbf{m}_U^2 + \mathbf{m}_D^2\right],$$

and the $\mathbf{m}_i^2$ are squared mass matrices in generation space. The numerical coefficients a_i, b_i and c_i are related to the quantum numbers of the scalar fields, but are all positive quantities. Incorporation of large, multi-TeV masses for the first and second generation scalars leads to an overall positive, *non-negligible* contribution to the slope of SSB mass trajectories versus energy scale [36]. Although formally a two loop effect, the smallness of the couplings is compensated by the multi-TeV scale values of masses for the first two generations of scalars. In running from M_{GUT} to M_{weak}, the two-loop terms result in an overall *reduction* of scalar masses, and its effect depends on the quantum numbers of the various scalar fields.

Generational non-degeneracy of scalar masses, especially for the 3GS scenario, is natural in SO(10) SUSY GUT models where above-the-GUT-scale-running is allowed. In this case, the unified third generation Yukawa coupling acts to suppress $m_{16}(3)$ with respect to $m_{16}(1,2)$, even if the three generations are degenerate at some higher scale, e.g. at the Planck scale M_P.

SO(10) model RGEs are presented in section 6.1 of ref. [21]. As an example, we show in figure 1 the evolution of m_{10}^2, $m_{16}(1,2)$ and $m_{16}(3)$ from M_P to M_{GUT} for model parameters as depicted in the caption. We see that a splitting of order 25% is possible at M_{GUT} for parameter choices as befitting Yukawa unified models. The natural splitting here is then that $m_{16}(3) < m_{16}(1,2)$, assuming degeneracy at M_P. In our following calculations, we will merely implement $m_{16}(3)$ as a free parameter as distinct from $m_{16}(1,2)$.
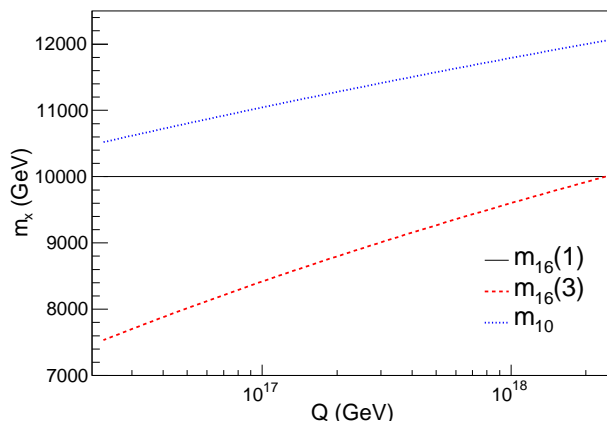
Figure 1. Renormalization group evolution of soft SUSY breaking terms $m_{16}(1,2)$, $m_{16}(3)$ and m_{10} from M_P to M_{GUT} in a simple SO(10) SUSY GUT. We take $g_{\text{GUT}} = 0.682$, $f_{\text{GUT}} = 0.56$, $m_{16} = 10\,\text{TeV}$, $m_{10} = 12.05\,\text{TeV}$ and $A_0 = -19.947\,\text{TeV}$ (point B of table 2 of ref. [14]).

3 Numerical results for Yukawa unified models

3.1 MCMC scan of parameter space

In this section, in addition to the just-so HS model, we will also scan over the SO(10) model parameter space as given by

$$m_{16}(1,2), \quad m_{16}(3), \quad m_{10}, \quad M_D, \quad m_{1/2}, \quad A_0, \quad \tan\beta, \quad sign(\mu), \tag{3.1}$$

where we also update our boundary conditions of $m_b(M_Z)^{\overline{DR}} = 2.416$, $\alpha_s(M_Z) = 0.118$ and $m_t = 173.1\,\text{GeV}$; the latter in accord with recent Tevatron measurements [37]. In addition, we augment the parameter space with the RHN parameter set

$$M_{N_3}, \quad f_{\nu_\tau}, \quad A_{\nu_\tau}, \quad m_{\tilde{\nu}_{R3}} \tag{3.2}$$

where we assume $f_\nu = f_t$ at $Q = M_{\text{GUT}}$, $A_\nu = A_0$ and $m_{\tilde{\nu}_{R3}}$ is as given in eq. (1.5) by DT splitting. For M_{N_3}, we are guided by the simple see-saw relation

$$m_{\nu_3} \simeq \frac{(f_{\nu_\tau} v_u)^2}{M_{N_3}} \tag{3.3}$$

where for $\tan\beta \sim 50$, we have $v_u(m_{\text{SUSY}}) \sim 171.6\,\text{GeV}$, $v_d(m_{\text{SUSY}}) \sim 3.5\,\text{GeV}$ and $f_{\nu_\tau} \sim 0.54$. Then, cosmological bounds on the sum of neutrino masses implies $\sum m_{\nu_i} \stackrel{<}{\sim} 1\,\text{eV}$. This implies $M_{N_e} \stackrel{>}{\sim} 10^{13}\,\text{GeV}$. We will adopt here $M_{N_3} = 10^{13}\,\text{GeV}$ to maximize the downward push of f_{ν_τ} consistent with bounds on neutrino masses. The superparticle mass spectrum is then generated using Isajet 7.79 [17], which includes full two-loop RGE running, implementation of the RG-improved 1-loop effective potential for EWSB, and full 1-loop corrections to all sparticle masses. We scan over the parameter space using the MCMC method with a Metropolis sampling algorithm as in [12], which provides an optimized search for parameter space points with the lowest R values.
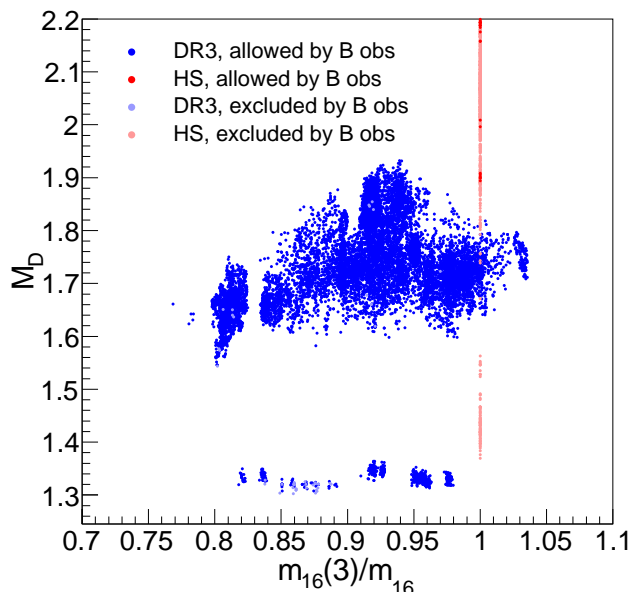
model	$R_{\min}$
mSUGRA	1.35
mSUGRA+RHN	1.19
SO(10)+RHN, $M_D = 0$	1.19
SO(10)+DT	1.08
SO(10)+3GS	1.30
SO(10)+DT+3GS	1.06
SO(10)+DT+RHN	1.04
SO(10)+RHN+3GS	1.17
SO(10)+DT+RHN+3GS (DR3)	1.025
SO(10)+HS	1.0

Table 1. Minimal R values obtained from MCMC scans for $t - b - \tau$ Yukawa unification in various model parameter space choices.

In the scans, we require that the mass limits from direct SUSY [38] and Higgs [39] searches at LEP be observed (additional limits from Tevatron searches do not affect the solutions with small R). Moreover, we take into account the constraints from the branching fractions for the $b \to s\gamma$ and $B_s \to \mu^+\mu^-$ decays. The measured branching ratio of the inclusive radiative B decay is $\mathrm{BR}(b \to s\gamma) = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$ [40], and the SM theoretical prediction $\mathrm{BR}(b \to s\gamma)^{\mathrm{SM}} = (3.15 \pm 0.23) \times 10^{-4}$ [41]. Combining experimental and theoretical errors in quadrature, we take $2.85 \le \mathrm{BR}(b \to s\gamma) \times 10^4 \le 4.19$ at 2σ together with the 95% CL upper limit $\mathrm{BR}(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-8}$ from CDF [42]. We adopt the IsaJet Isatools [43] program for the calculations of $\mathrm{BR}(b \to s\gamma)$ and $\mathrm{BR}(B_s \to \mu^+\mu^-)$.

Our results for parameter space scans with minimized R values are summarized in table 1. We first see that $t - b - \tau$ Yukawa unification is not possible in the mSUGRA model: in this case, $R_{\min} = 1.35$. However, adopting SO(10) parameter space with RHN parameters (but with $M_D = 0$) actually allows considerable improvement with R values as low as 1.19 to be attained. The same $R_{\min}$ can be obtained in mSUGRA+RHN. Allowing SO(10)+DT splitting (but no RHN), we find $R_{\min} = 1.08$, while SO(10)+HS yields $R_{\min} = 1.0$, as noted in many previous studies. An allowance of DT+3GS yields a value $R_{\min} = 1.06$ while allowing DT+RHN gives $R_{\min} = 1.04$. SO(10) scans with DT+RHN+3GS (the DR3 model) allow us to generate models with $R_{\min}$ values down to 1.025. The remaining few per cent may be accounted for either by GUT scale threshold corrections, or theoretical error due to our imperfect modeling of Yukawa coupling boundary conditions and evolution over $13 - 14$ orders of magnitude. If these combined effects are at the level of few per cent, then the SO(10) model with D-term splitting plus RHN and/or 3GS can be seen to be in accord with Yukawa unified models for $\mu > 0$. In the following, we concentrate on the DR3 model, which combines DT+RHN+3GS and gives the best prospects for Yukawa unification, and compare it to the "just-so" Higgs splitting.

Next, we examine which parameter choices lead to $t - b - \tau$ Yukawa coupling unification in the DR3 model. To this end, we plot the locus of Yukawa unified solutions with $R < 1.05$
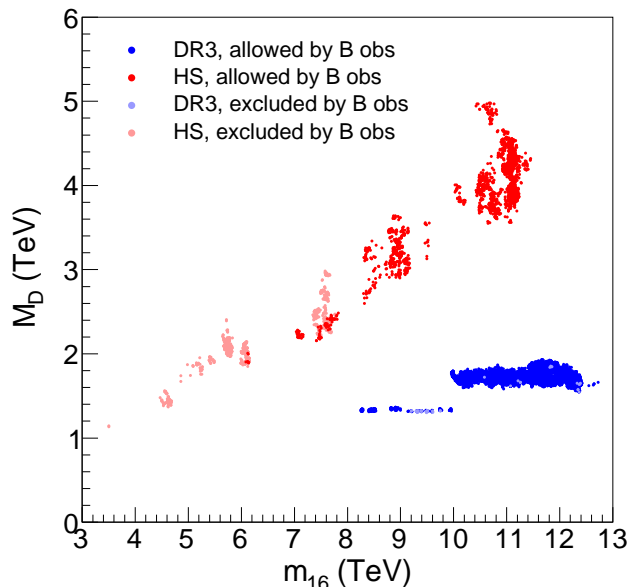
Figure 2. Yukawa unified solutions with $R < 1.05$ from "just-so" HS model (red points) and the DR3 model (blue points) in the $m_{16}(3)/m_{16}$ *vs.* M_D plane. Light blue/light red points are excluded by B-physics constraints. Note that M_D extends up to about 5 TeV in the HS case, cf. figure 3.

in figure 2 in the $m_{16}(3)/m_{16}$ *vs.* M_D plane.[4] Here and in the following, $m_{16} \equiv m_{16}(1,2)$ for simplicity. The red points indicate results from the just-so HS model. They necessarily all have $m_{16}(3)/m_{16} = 1$, and so form a vertical line through the plot. The values of M_D in the HS model actually range up to ~ 5 TeV. In contrast, we find that the DR3 model yields Yukawa-unified solutions provided that $m_{16}(3) \sim (0.8 - 1.05)m_{16}(1,2)$: i.e. the third generation scalars are favored to be at somewhat lower GUT scale masses than their first/second generation counterparts. In addition, we see that M_D is now restricted to lie in the $1 - 2$ TeV range: significantly less than the splitting needed for the HS model.

In figure 3, we show the locus of points with $R < 1.05$ in the M_D *vs.* m_{16} plane. Here we see that for the HS model, values of $M_D \sim 0.33m_{16}$ are needed. In the case of the DR3 model, we see that the value of M_D needed also grows with m_{16}. But in this case, we find instead that $M_D \sim 0.13m_{16}$. The amount of Higgs splitting needed in the DR3 model is much less than in the HS model since additional Higgs splitting comes from the effect of the RHN and 3GS. Other than the relative reduction in M_D needed for a given value of $m_{16}(1,2)$, the usual SSB mass relations for Yukawa-unified HS model still hold.

Note also that in the DR3 model, Yukawa-unified solutions occur only for $m_{16}(1,2) \overset{>}{\sim}$ 8 TeV, while in the HS model they also occur for smaller values of m_{16}. However, the HS points with $m_{16} \sim 4 - 8$ TeV are almost all excluded by BR($b \to s\gamma$) and/or BR($B_s \to \mu^+\mu^-$). This observation is also made by Altmannshofer et al. [13]. The DR3 model points, on the other hand, are much less affected by the B-physics constraints. We will come back to this in section 3.3.

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[4]Note that in our case M_D^2 is always positive since we need $m_{H_u}^2 < m_{H_d}^2$ at the GUT scale.
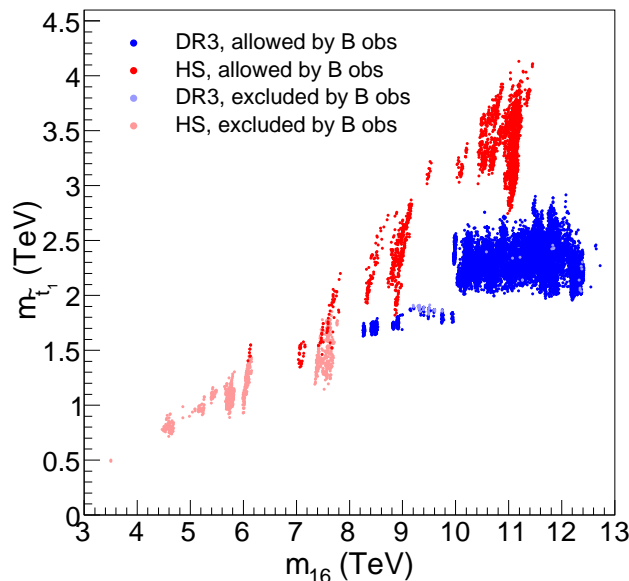
Figure 3. Yukawa unified solutions with $R < 1.05$ from "just-so" HS model (red points) and models with DR3 splitting (blue points) in the M_D vs. m_{16} plane; points excluded by B-physics constraints are shown in lighter colour.

3.2 SUSY particle mass spectrum

Given that both the HS and DR3 models lead to $t - b - \tau$ unification at M_{GUT}, the next question is whether it is possible to physically distinguish between these models at experiments. Several differences between the SUSY particle mass spectrum lead us to believe that the models are at least in principle distinguishable. The first point is that — in the case of the DR3 model — the GUT scale soft masses for m_D^2 and m_L^2 are diminished by $-3M_D^2$ relative to the value of m_{16}. The first and second generation values of m_D and m_L are expected to be in the multi-TeV regime, and so their mass diminution by D-terms isn't likely to be visible at any collider expected to operate in the near future. However, the third generation scalar masses are driven to weak scale values by the RIMH mechanism, and are expected to be in the 1-2 TeV regime. Thus, we would expect the third generation $\tilde{b}_R$ and $\tilde{\tau}_L$ masses to be diminished with respect to expectations from the HS model. This effect should be most noticable in the bottom squark sector, since in the tau slepton sector, we usually expect (based on the form of the MSSM RGEs) $m_{\tilde{\tau}_L} > m_{\tilde{\tau}_R}$, whereas in the sbottom sector, we expect $m_{\tilde{b}_R} < m_{\tilde{b}_L}$.

In figure 4, we show the value of $m_{\tilde{t}_1}$ vs. m_{16} for Yukawa-unified models with $R < 1.05$ in the HS (red dots) and DR3 (blue dots) cases. Points which obey the mass limits but do not pass B-physics constraints are again shown in lighter colour. We see that for a given value of m_{16}, the value of $m_{\tilde{t}_1}$ is smaller in the DR3 case than in the HS case. Naively, one might expect the opposite result, since in the DR3 model both $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$ are *increased* by $+M_D^2$. However, two effects act counter to the D-term. First, there is the third generation mass splitting, which typically reduces $m_{16}(3)$ relative to $m_{16}(1)$. Second, there is an additional contribution to RG running of top squark soft masses from the S-
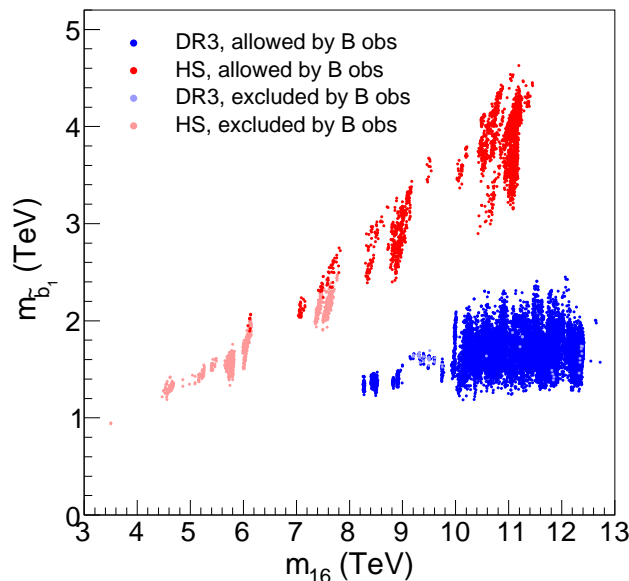
Figure 4. Yukawa unified solutions with $R < 1.05$ from "just-so" HS model (red points) and models with DR3 splitting (blue points) in the m_{16} $vs.$ $m_{\tilde{t}_1}$ plane; points excluded by B-physics constraints are shown in lighter colour.

term, eq. (2.6), which is zero for models with strict universality, but which is non-zero for models with Higgs mass splitting. For our case with $m^2_{H_u} < m^2_{H_d}$, the S-term gives an *upwards* push to the top squark soft mass evolution. Since the Higgs splitting is much less in the DR3 model, there is a smaller upwards push from the S-term, and this effect coupled with 3GS wins out over the increased mass due to the D-term, thus giving the DR3 models typically a smaller $\tilde{t}_1$ mass than in the HS case. We note here that while the value of $m_{\tilde{t}_1}$ is smaller in the DR3 case compared to HS– for a given value of m_{16}– the value of m_{16} will not be easily measureable, and so the $m_{\tilde{t}_1}/m_{16}$ ratio is not likely a good discriminator between the two models: for instance, if a value of $m_{\tilde{t}_1} \sim 1.5\,\mathrm{TeV}$ is found at some future experiment, it will be difficult to determine if it is consistent with the HS model with $m_{16} \sim 7\,\mathrm{TeV}$, or with the DR3 model with $m_{16} \sim 9\,\mathrm{TeV}$.

In figure 5, we plot the value of $m_{\tilde{b}_1}$ expected from Yukawa-unified models with HS (red dots) and DR3 (blue dots), where again we only plot solutions with $R < 1.05$. We see that in the HS case, for a given value of m_{16}, the value of $m_{\tilde{b}_1}$ is always lowest in the DR3 case. In fact, for $m_{16} \sim 10\,\mathrm{TeV}$, we expect $m_{\tilde{b}_1} \sim 1-2\,\mathrm{TeV}$, while in the HS case, $m_{\tilde{b}_1} \sim 3-4\,\mathrm{TeV}$.

The mixing angle of $\tilde{b}_1$ is also strongly affected. Here, following the notation of ref. [44], we have $\tilde{b}_1 = \cos\theta_b \tilde{b}_L - \sin\theta_b \tilde{b}_R$. As an illustrative example, we list in table 2 the spectrum from a HS Yukawa-unified point with $m_{16} = 10\,\mathrm{TeV}$, and a DR3 model case with $m_{16} = 11.8\,\mathrm{TeV}$. The evolution of all four Yukawa coupings are shown for the DR3 point in figure 6.

In the HS case, $\tilde{b}_1$ is $\sim 10\%$ $\tilde{b}_R$, while in the DR3 case, $\tilde{b}_1$ is 99.8% $\tilde{b}_R$. This comes from the fact that the D-term mass contribution pushes $m_{\tilde{b}_L}$ up, and $m_{\tilde{b}_R}$ down. We also show in figure 7 the b-squark mixing angle θ_b versus $m_{\tilde{b}_1}$. In this plot, we see the value of $\theta_b \sim 1.5$ in the DR3 case, which means the $\tilde{b}_1$ is dominantly $\tilde{b}_R$. Meanwhile, the red points indicate that in the HS model, the $\tilde{b}_1$ is dominantly $\tilde{b}_L$.
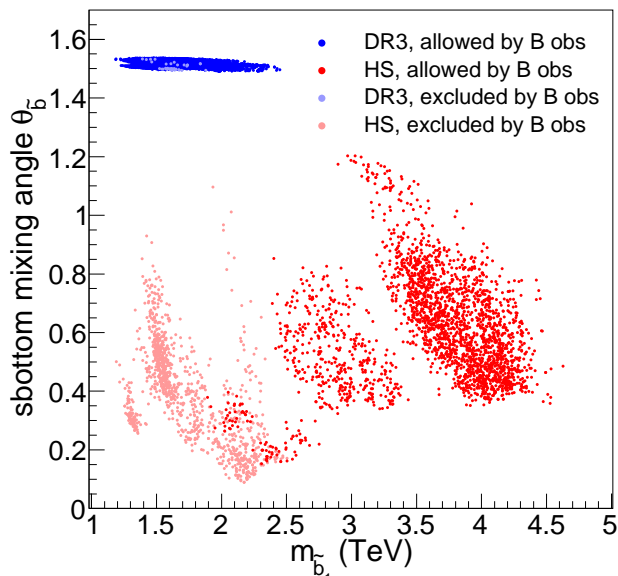
Figure 5. Yukawa unified solutions with $R < 1.05$ from the "just-so" HS model (red points) and models with DR3 splitting (blue points) in the m_{16} *vs.* $m_{\tilde{b}_1}$ plane; points excluded by B-physics constraints are shown in lighter colour.
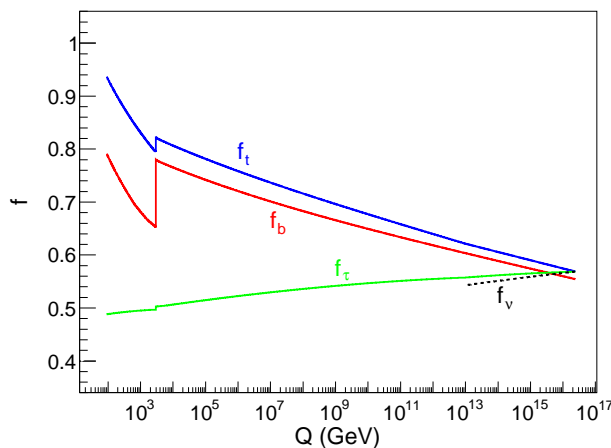
Figure 6. Evolution of all four Yukawa couplings for the case of point DR3 listed in table 2.

The composition of $\tilde{b}_1$ in principle might be measureable at the LHC if a bottom squark production event sample can be isolated, and the $\tilde{b}_1$ branching fractions can be measured. Here note that the $\tilde{b}_L$ decays into wino-like charginos and neutralinos and/or into $W\tilde{t}_1$, while the $\tilde{b}_R$ does not. To give a concrete example, we compare the $\tilde{b}_1$ decays at the DR3 point of table 2 to those of a HS point with a very similar $\tilde{b}_1$ mass: point A of ref. [14]. This latter point has $m_{16} = 5\,\text{TeV}$, $m_{\tilde{b}_1} = 1322\,\text{GeV}$, $m_{\tilde{t}_1} = 834\,\text{GeV}$, $m_{\tilde{g}} = 363\,\text{GeV}$, $m_{\tilde{\chi}_1^\pm, \tilde{Z}_2} = 109\,\text{GeV}$ and $m_{\tilde{Z}_1} = 50\,\text{GeV}$. Several relevant $\tilde{b}_1$ branching fractions are listed in table 3. As can be seen, in the DR3 model, $\tilde{b}_1$ decays nearly 100% of the time into $b\tilde{g}$, while in the HS model, there is a sizable branching into $t\tilde{\chi}_1^-$ and $W\tilde{t}_1$ states.

Figure 7. Yukawa unified solutions with $R < 1.05$ from the "just-so" HS model (red points) and models with DR3 splitting (blue points) in the $m_{\tilde{b}_1}$ $vs.$ θ_b (rad.) plane; points excluded by B-physics constraints are shown in lighter colour.

While it is conceivable that the $L - R$ composition of $\tilde{b}_1$ might be measured at LHC, the measurement would likely be very difficult and intricate. However, a measurement of the composition of $\tilde{b}_1$ would likely be quite straightforward at a linear e^+e^- collider with sufficient energy to produce $\tilde{b}_1\bar{\tilde{b}}_1$ pairs. First, the branching fractions of $\tilde{b}_1$ would likely be much easier to dis-entangle at an e^+e^- collider than at the LHC. Second, a linear e^+e^- collider is likely to be constructed with polarizable electron beams. The total $e^+e^- \to \tilde{b}_1\bar{\tilde{b}}_1$ cross section will be very sensitive to the beam polarization, and the composition of the $\tilde{b}_1$. The beam polarization-dependent sparticle pair production cross sections have been calculated in ref. [45], and the results are plotted versus the beam polarization $P_L(e^-)$ in figure 8. Here, we see that $\sigma(e^+e^- \to \tilde{b}_1\bar{\tilde{b}}_1)$ peaks at $P_L(e^-) = +1$ in the case of the HS model, whereas just the opposite peak at $P_L(e^-) = -1$ is expected in the DR3 case.

In figure 9, we show the value of m_A $vs.$ m_{16} for Yukawa-unified models with $R < 1.05$ in the HS and DR3 cases. For moderate to large $\tan\beta$, at tree level we roughly expect that $m_A^2 \sim m_{H_d}^2 - m_{H_u}^2$, i.e. that the value of m_A^2 is nearly equal to the weak scale mass splitting between the two Higgs soft masses. In the DR3 model, where a much smaller Higgs splitting is needed at the GUT scale in order to accomplish REWSB, we also find a significantly smaller value of m_A expected for a given value of m_{16}. For $m_{16} \sim 10\,\text{TeV}$, we typically get $m_A \sim 1\,\text{TeV}$ in the DR3 model, while $m_A \sim 3\,\text{TeV}$ in the HS model.

The value of m_A may be readily established at the LHC, especially for the large $\tan\beta \sim 50$ case expected for Yukawa-unified models. In the large $\tan\beta$ case, the b-quark Yukawa coupling is large, and production cross sections for A are enhanced, both via glue-glue fusion (triangle diagrams) and via bA and $b\bar{b}A$ production. Then the A is typically expected to decay into modes such as $b\bar{b}$, $\tau^+\tau^-$ and $\mu^+\mu^-$. The first two of these offer a rough mass bump

parameter	Pt. B [14]	DR3
$m_{16}(1,2)$	10000	11805.6
$m_{16}(3)$	10000	10840.1
m_{10}	12053.5	13903.3
M_D	3287.1	1850.6
$m_{1/2}$	43.9442	27.414
A_0	-19947.3	-22786.2
$\tan\beta$	50.398	50.002
R	1.025	1.027
μ	3132.6	2183.4
$m_{\tilde{g}}$	351.2	321.4
$m_{\tilde{u}_L}$	9972.1	11914.2
$m_{\tilde{t}_1}$	2756.5	2421.6
$m_{\tilde{b}_1}$	3377.1	1359.5
$m_{\tilde{e}_R}$	10094.7	11968.5
$m_{\tilde{\chi}_1^\pm}$	116.4	114.5
$m_{\tilde{Z}_2}$	113.8	114.2
$m_{\tilde{Z}_1}$	49.2	46.5
m_A	1825.9	668.3
m_h	127.8	128.6
θ_b (radians)	0.329	1.53

Table 2. Masses in GeV units and parameters for Yukawa-unified point B of ref. [14] with just-so HS, and a point with the DR3 model using $M_N = 10^{13}$ GeV. We also give the b-squark mixing angle.

parameter	Pt. A [14]	DR3
$m_{\tilde{b}_1}$	1321.8	1359.5
$\tilde{b}_1 \to t\tilde{\chi}_1^-$	8%	0.1%
$\tilde{b}_1 \to W\tilde{t}_1$	30%	–
$\tilde{b}_1 \to b\tilde{g}$	55%	99%

Table 3. Sbottom mass and branching fractions for a HS model (point A of ref. [14]) and for the DR3 point from table 2. Note that the two points have very similar $m_{\tilde{b}_1}$, but the $\tilde{b}_1$ is mainly a left-squark in the HS case, while it is mainly a right-squark in the DR3 case.

with which to reconstruct m_A; the latter mode into $\mu^+\mu^-$ suffers from a small branching fraction, but may offer a sharper mass bump since all the decay products are easily detected [46]. However, for extremely high energy muons, the momentum resolution — determined by track bending in the detector magnetic field — gets more difficult at high energies. Evaluation of the A mass bump in all three of these modes may allow good resolution on the A mass reconstruction. If a measurement of $A \to \mu^+\mu^-$ is possible, then it may also be possible to extract information on the width Γ_A, which is very sensitive to the value of $\tan\beta$.
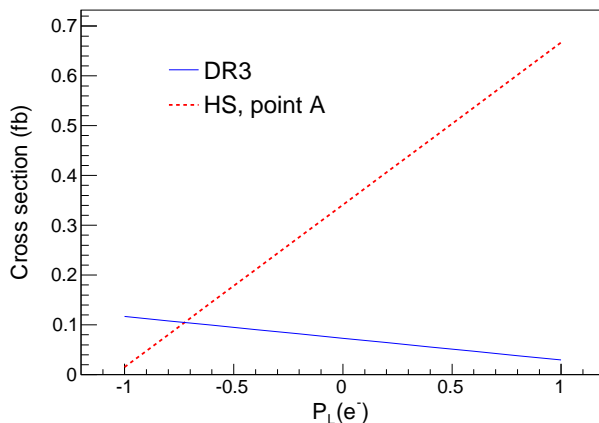
Figure 8. Sbottom production cross section, $\sigma(e^+e^- \to \tilde{b}_1\bar{\tilde{b}}_1)$, in fb versus electron beam polarization $P_L(e^-)$ for the CLIC accelerator with $\sqrt{s} = 3\,\mathrm{TeV}$. The blue curve corresponds to a DR3 model point while the red curve corresponds to the HS model point A of ref. [14].
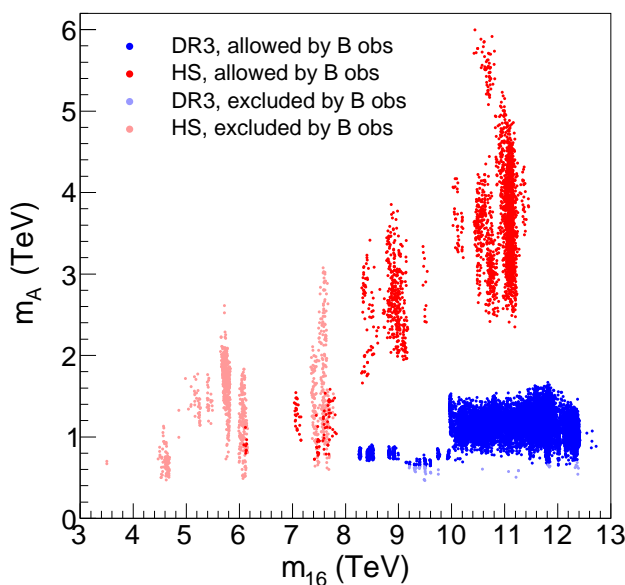

Figure 9. Yukawa unified solutions with $R < 1.05$ from "just-so" HS model (red points) and models with DR3 splitting (blue points) in the m_{16} *vs.* m_A plane; points excluded by B-physics constraints are shown in lighter colour.

3.3 Predictions for B-physics observables

For completeness, we present in figure 10 the explicit results for the branching ratios of the $b \to s\gamma$ and $B_s \to \mu^+\mu^-$ decays for the Yukawa-unified solutions with $R < 1.05$ of the previous section.

For the HS model, we see that most of the solutions with $m_{16} \stackrel{<}{\sim} 8\,\mathrm{TeV}$ lead to too low a value of BR($b \to s\gamma$), and hence are excluded. Of the remaining HS points with $m_{16} \stackrel{<}{\sim} 8\,\mathrm{TeV}$, a large fraction has too high a BR($B_s \to \mu^+\mu^-$). In the end, only a few points with $m_{16} \stackrel{<}{\sim} 8\,\mathrm{TeV}$ survive and they have BR($B_s \to \mu^+\mu^-$) $= \mathcal{O}(10^{-8})$. The
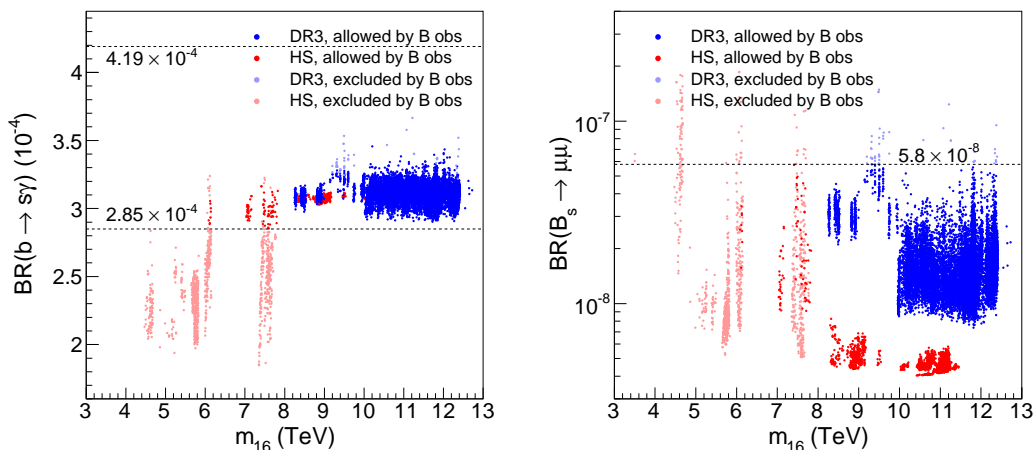
Figure 10. Predictions for $\text{BR}(b \to s\gamma)$ *vs.* m_{16} (left plot) and $\text{BR}(B_s \to \mu^+\mu^-)$ *vs.* m_{16} (right plot) from Yukawa-unified models in both the HS (red dots) and DR3 (blue dots) models. Also shown are the 2σ limits used in our analysis.

HS points with $m_{16} > 8\,\text{TeV}$ mostly comply with the B-physics constraints and have $\text{BR}(B_s \to \mu^+\mu^-) = \mathcal{O}(10^{-9})$.

The situation is different for the DR3 model points, for which m_{16} is always greater than 8 TeV. These points cluster around $\text{BR}(b \to s\gamma) \sim 3 \times 10^{-4}$, with only a very small fraction (less than 1%) excluded by a too high $\text{BR}(B_s \to \mu^+\mu^-)$. Indeed the excluded points are those with the lowest m_A values, cf. figure 9. Interestingly, owing to the lower m_A, the DR3 model predicts $\text{BR}(B_s \to \mu^+\mu^-) = \mathcal{O}(10^{-8})$, which may well be probed in the near future at the Fermilab Tevatron collider.

4 Summary and conclusions

In this paper, we have re-investigated the issue of electroweak symmetry breaking in Yukawa-unified SUSY models. It is well known that perfect Yukawa coupling unification can be achieved in simple SO(10) SUSY GUTs with universal soft-breaking terms for gauginos and sfermions, but non-universal Higgs-mass terms. However, it is hard to understand how this "just-so" Higgs splitting may arise only in the Higgs multiplets, and not in the corrsponding matter multiplets, as is expected in D-term splitting. We have found that Yukawa coupling unification good to few per cent can be achieved in the case of D-term splitting but only if one also allows for the presence of the neutrino Yukawa coupling, along with first/second versus third generation matter scalar splitting. Each of these three features — D-term splitting, right hand neutrino Yukawa couplings and third generation splitting, together comprising the DR3 model — are to be expected in simple GUT models based on SO(10). In this case, the DR3 model may be considered more satisfying from the SO(10) GUT point of view than the HS model.

The two models lead to many similarities in the expected sparticle mass spectra, but also some important differences. Regarding similarities, both lead to a split spectra with first/second generation scalars in the $\sim 10\,\text{TeV}$ regime, with third generation and heavy

Higgs scalars in the few TeV regime, along with a very light spectrum of gauginos. In particular, with gluinos expected in the $300 - 500\,\mathrm{GeV}$ mass range, a robust variety of gluino pair production events are expected at the CERN LHC [47]. Regarding differences, the amount of Higgs splitting for a given value of m_{16} is expected to be much less in the DR3 model, leading to much lighter values of m_A, m_H and $m_{H^\pm}$. These heavier Higgs states stand a much higher chance to be detectable at LHC in the DR3 model, as compard to the HS model. In addition, the lightest bottom squark is expected to be much lighter for a given value of m_{16} in the DR3 model, as compared to the HS model. Also, the smoking gun difference is that the $\tilde{b}_1$ should be predominantly a right-squark in DR3, while it is expected to be dominantly a left-squark in the HS model. This can in principle be detected at LHC due to the different $\tilde{b}_1$ branching fractions which are expected; however, in practise, this differentiation is likely to be a difficult enterprise. It will be much simpler at a CLIC-type e^+e^- linear collider operating with $\sqrt{s} > 2m_{\tilde{b}_1}$. In this case, the $\tilde{b}_1$ production and decay modes should be more readily identified, and especially the total $\tilde{b}_1\bar{\tilde{b}}_1$ production cross section will depend on electron beam polarization in very different fashions for the two models. In this case, the two models should then be easily distinguishable.

We also evaluated predictions for $\mathrm{BR}(b \to s\gamma)$ and $\mathrm{BR}(B_s \to \mu^+\mu^-)$ in the HS and DR3 models. In the HS model, most points with $m_{16} < 8\,\mathrm{TeV}$ are excluded by these constraints. In the case of the DR3 model, which apparently requires $m_{16} \stackrel{>}{\sim} 8\,\mathrm{TeV}$, 99% of the points are allowed by the B-decay constraints. But the predicted rate for the $B_s \to \mu^+\mu^-$ decay is just below its current experimental limit, and may well be probed in the near future as data continues to accrue at the Fermilab Tevatron collider.

Finally we note that a large RHN Yukawa coupling, as assumed in this study, can lead to lepton flavor violation (LFV) by generating off-diagonal LFV terms in the charged-slepton mass matrix through RG running [48, 49]. This can lead to additional constraints which may further sharpen the predictions for Yukawa-unified models. This issue is left for future work.

Acknowledgments

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